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A hybrid technique for the efficient reliability computation of large structures

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ABSTRACT

In the case of large structures, one obstacle encountered when the computation of reliability is attempted by most methods, is the presence of large numbers of random variables. Another issue is that computing the reliability of large structures typically requires conducting their structural analysis, which can be computationally time consuming, a large number of times. These concerns may render the reliability analysis computationally expensive or even unachievable. In this paper, a hybrid technique for computing the reliability of large structures is presented. In this technique, the most probable point of failure (MPP) is determined first using modified concepts of the Weighted Average Simulation Method (WASM). The WASM concepts are modified to handle the problem of large random variables present in large structures and also in order to find the MPP in a computationally more efficient manner. Once the MPP is determined, it is transformed into the standard normal space. Hence, the reliability index is calculated in closed-form in the standard normal space. The approach is tested on a truss bridge example.

1. Introduction

The aim of computing structural reliability is to assess the safety of an engineered system by considering how its pre-specified performance is affected by random variations and uncertainties in demands, system properties, boundary and initial conditions, etc. (Bichon et al., 2011; Haukaas and Der Kiureghian, 2007). Existing structures are typically large, where they are constructed of many members, and accordingly a large number of associated random variables are encountered in the reliability analysis of the structure. For large structures, requiring time-consuming structural analysis, component-level reliability analysis can be a computationally expensive task and system-level reliability analysis can be even more so (Bichon et al., 2011). The computational cost is exacerbated when the reliability analysis is embedded into a design optimization application. However, the crucial importance of such goal calls for the development of computationally efficient techniques for conducting the reliability analysis of large structures.

In the literature, two classes of methods devoted for the computation of structural reliability are dominant. These two classes are the Most Probable Point (MPP)-based methods and the simulation-based class of methods. The following is a brief review of these two classes. As will be shown in later sections, the hybrid technique presented in this paper is a cross-over between methods from both of these classes.

The MPP is the point that has the highest likelihood among all points in the failure region (Der Kiureghian and Dakessian, 1998; Rahman and Wei 2006). In other words, it is the point that has the highest probability density on the limit state function. The most widely used method in the MPP-based class is the First Order Reliability Method (FORM) (Ditlevsen and Madsen, 1996). According to this method, the performance function is approximated by a hyperplane which is tangent to the failure surface at the MPP. Additionally, the Second Order Reliability Method (SORM) approximates the performance function by a quadratic hypersurface in the neighborhood of the MPP. Several other MPP-based methods were also proposed in the literature (Breitung, 1984; Tvedt 1990; Der Kiureghian et al., 1987; Der Kiureghian and De Stefano, 1991; Hong et al., 1999; Zhao and Ono, 2001; Xu and Cheng, 2003; Adhikari, 2004, Hohenbichler et al., 1987; Koyluoglu and Nielsen, 1994; Hasofer and Lind, 1974; Wu et al., 1990). Conducting these methods typically involves the calculation of the performance function gradients with respect to the random variables present in the

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problem. Performance functions in large structures are usually implicit functions that require conducting structural analysis operations. Thus, gradients are determined numerically in such cases, which requires the evaluation of the performance function several times for each random variable. As the number of random variables increases in a problem, such as in the typical case of large structures, the required number of performance function evaluations increases accordingly. Hence, the computational efficiency of these methods becomes in question when problems with large numbers of random variables are encountered.

The most known simulation-based technique is the Monte Carlo Simulation (MCS), (Melchers, 1999). MCS is inefficient if the evaluation of the performance function is computationally expensive or if the probability of failure is very small (Choi et al., 2006; Melchers, 1999; Nowak and Collins, 2013). Indeed, the repetitive structural analysis of large systems can be computationally cumbersome. Researchers have developed numerous other simulation-based methods over the years, including stratified sampling (Ziha, 1995), Latin Hypercube Sampling (Olsson et al., 2003; Huntington and Lyrintzis, 1998), Importance Sampling (Melchers, 1990; Ibrahim, 1991), Response Surface Methods (Allaix and Carbone, 2011; Rajashekhar and Ellingwood, 1993), Directional Sampling (Ditlevsen et al., 1990; Melchers, 1994; Nie and Ellingwood, 2000), Subset Simulation (Au et al., 2007; Miao and Ghosn, 2011), Line Sampling (Pradlwarter et al., 2007; Lu et al., 2008; Depina et al, 2016) and Local Domain Monte Carlo simulation (Pradlwarter and Schuëller, 2010). The Weighted Average Simulation Method (WASM) (Rashki et al., 2012; 2014a, b; Luo et al., 2014) is one of the most recent methods which have been proven to be capable of determining the reliability with a reasonably small number of generated samples.

In this paper, a hybrid technique for computing the reliability of large structures is presented. In this technique, the MPP is determined first using modified concepts of the WASM. The WASM concepts are modified in order to find the MPP more efficiently such that the performance function is evaluated for a small portion of the generated samples. It is also shown that the WASM faces a numerical problem when large numbers of random variables are handled in the problem. This problem is overcome in this paper by efficiently modifying the weight index equation. Once the MPP is determined, it is transformed into the standard normal space. Hence, the reliability index is calculated in closed-form in the standard normal space. The approach is tested on a bridge example.

2. Structural reliability

Determination of the structural reliability requires the establishment of the performance function. The performance function, g, associated with failure of a component having resistance R and subjected to load L can be calculated by:

$$G(R,L) = R - L \tag{1}$$

A limit state function is the performance function satisfying the condition g = 0. The probability of failure can be defined as the chance that a particular combination of R and L will give a negative value of g, which can be expressed as:

$$P_f = \text{probability that } [g < 0] \tag{2}$$

Structural reliability is typically measured by the reliability index, β, which can be determined as:

$$\beta = \Phi^{-1}(1 - P_f) \tag{3}$$

where ϕ^{-1} is the inverse of the standard normal distribution function.

3. Determination of the MPP by WASM

The WASM is a simple yet efficient and robust method that is capable of overcoming the limitations and difficulties of current reliability methods. The method can be used to determine the probability of failure and MPP. In this paper, emphasis is given to the determination of the MPP. One of the advantages of using the WASM in finding the MPP is that an explicit performance function is not needed. Another advantage is that there is no concern of risking convergence to a local MPP. The procedure for determining the MPP by the WASM is summarized in the following steps:

1- Proper intervals for each random variable in the problem are determined. Rashki et al. (2012) suggested that an MCS can be used to determine the upper and lower points for the interval of each random variable.
2- Samples for all random samples are generated in a random variable space. The uniform distribution can be used for generating these random samples (Rashki et al., 2012).
3- A weight index is determined for each sample as the product of the probability density functions (PDFs) of the variables as follows (Rashki et al. 2012):

$$W(i) = \prod_{j=1}^{n} f_j(i) \tag{4}$$

where W(i) is the weight index of the i^{th} sample, f_j is the PDF of the j^{th} variable, and n is the number of random variables.

4- The index function, I(i), for the i^{th} sample is then established by evaluating the performance function and thus I(i) is determined as (Rashki et al., 2012):

$$I(i) = \begin{cases} 1 & \text{for } g_i < 0 \\ 0 & \text{for } g_i \geq 0 \end{cases} \tag{5}$$

5- In the WASM, the MPP is the point in the failure region with the highest weight index since it is considered as the point with the highest failure potential (Rashki et al., 2012). The MPP can thus be determined as follows:

$$\text{MPP} = \max_{i=1}^{N} \{I(i) \cdot W(i)\} \tag{6}$$

where N is the number of samples.

4. Determination of the probability of failure by WASM

According to the WASM, the probability of failure is defined as the ratio of the summation of weight indices in the failure region to the summation over the entire region. After performing steps 1 through 4 in the above procedure, the probability of failure can now be determined as follows (Rashki et al., 2012):

$$P_f = \frac{\sum_{i=1}^{N} I(i) \cdot W(i)}{\sum_{i=1}^{N} W(i)} \tag{7}$$

5. Calculation of the reliability index in standard normal space

The reliability index can be defined as the minimum distance between the origin and the failure surface in standard normal space (Hasofer and Lind, 1974). Fig. 1(a) shows a hypothetical space for random variables X_1 and X_2 along with a limit state function $g(X_1, X_2) = 0$. In order to determine the reliability index according to Hasofer and Lind (1974), the random variables and performance function are transformed to the standard normal space as shown in Fig. 1(b).

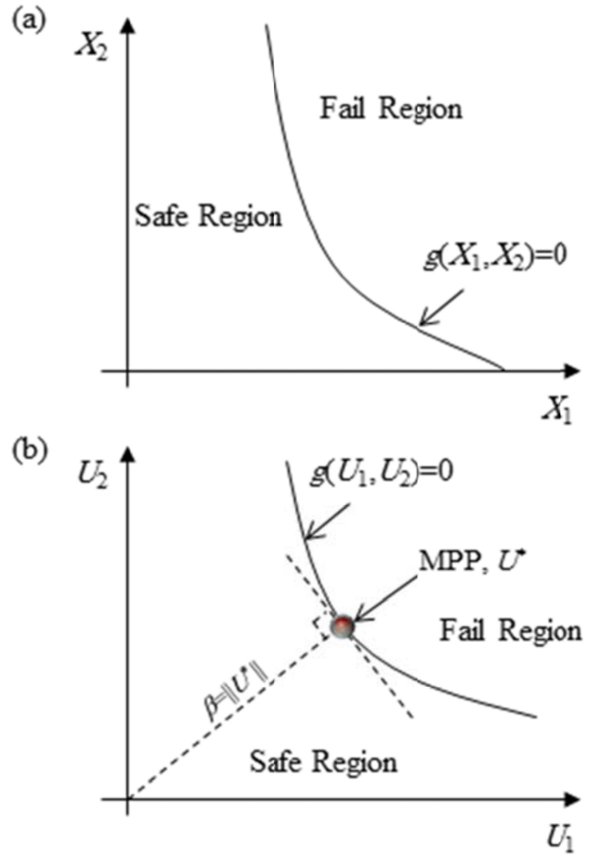

Fig 1: Limit state function, $g = 0$, in (a) space of random variables X_1 and X_2, and (b) standard normal space U_1 - U_2

The transformation of n normally distributed random variables $\mathbf{X} = [X_1, X_2, ..., X_n]$ into a set of standard variables $\mathbf{U} = [U_1, U_2, ..., U_n]$ with zero means and unit covariance matrix can be achieved by applying the following equation (Der Kiureghian and Liu, 1985):

$$\mathbf{U} = \mathbf{\Gamma} \mathbf{D} (\mathbf{X} - \mathbf{M}) \tag{8}$$

where $\mathbf{D} = \text{diag}[\sigma_i]$ = the diagonal matrix for the standard deviations such that σ_i is the standard deviation of variable X_i, $\mathbf{M} = [m_1, m_2, ..., m_n]^T$ = the mean vector such that m_i is the mean of variable X_i, and $\mathbf{\Gamma} = \mathbf{L}^{-1}$, where $\mathbf{L}$ is the lower-triangle matrix obtained from Cholesky decomposition (Melchers, 1999) of the correlation matrix $\mathbf{R}$ which is equal to (Der Kiureghian and Liu, 1985):

$$\mathbf{R} = \mathbf{D}^{-1} \mathbf{C} \mathbf{D}^{-1} \tag{9}$$

where $\mathbf{C} = [\rho_{ij}\sigma_i\sigma_j]$ = the covariance matrix such that ρ_{ij} is the correlation coefficient of variables X_i and X_j. The performance function can be transformed into the standard space as follows (Der Kiureghian and Liu, 1985):

$$g(\mathbf{U}) = g(\mathbf{M} + \mathbf{D}\mathbf{L}\mathbf{U}) \tag{10}$$

As shown in Fig. 1(b), the MPP is the closest point on the limit state function to the origin in the standard normal space, and its coordinates are $\mathbf{U}^* = [U_1^*, U_2^*, \dots, U_n^*]$. The reliability index is the Euclidean distance from this point to the origin. The MPP in the original space is $\mathbf{X}^*$. Accordingly, the reliability index can be calculated as follows:

$$\beta = \|\mathbf{U}^*\| = \sqrt{\mathbf{U}^* \cdot \mathbf{U}^*} \tag{11}$$

6. The problem of floating-point arithmetic in the weight index

The calculation of the weight index involves values of the density function, where in most cases its value is a small fraction. This is especially the case when the sample that is generated from the random variable lies in the vicinity of the tail of the distribution. As Equation 4 shows, the weight index for a sample generated from multiple random variables is the multiplication of these possibly small fractions. Of course, as the number of random variables grows, the result of this multiplication gets smaller and smaller. However, one must be aware of the computational capabilities and the limits to which small numbers can be handled by computer software.

In modern mathematical software, such as MATLAB (MathWorks, 2015), floating-point numbers are typically represented by double precision format, which requires 64 bits of storage. Based on this process, there is a smallest and largest number that can be represented with this format. In MATLAB, the smallest decimal fraction is about $d_{min} = 2.22507 \times 10^{-308}$ (MathWorks, 2015). However, and as will be shown later in this paper, values of the weight index can be a decimal fraction smaller than d_{min}, especially when large numbers of random variables are handled. Decimal fractions smaller than d_{min} are assigned the value of zero in MATLAB (MathWorks, 2015). Accordingly, this numerical issue will prevent the WASM from providing a valid solution.

In order to overcome this issue and make the proposed technique capable of determining the reliability of problems with large numbers of random variables, it is proposed in this paper that Equation 4 for calculating the weight index is modified as follows. In the proposed technique that will be described in detail in the next section, one is only interested in calculating the weight index of the samples and comparing between their values. Accordingly, it is mathematically permitted to transform Equation 4 through a strictly increasing function. The purpose of this transformation, of course, is to somehow change the scale of the small weight index values without distorting the weight index comparison outcomes between the samples. The natural logarithm function, ln(x), has properties that are perfectly suited for the purposes of the proposed technique. Fig. 2 shows the values of ln(x) as x changes from 10^{-300} to 10^{300}. The ln(x) values that correspond to 10^{-300} and 10^{300} are -690.8 and 690.8, respectively. Furthermore, it is known that the natural logarithm function has the following property:

$$\ln(xy) = \ln(x) + \ln(y) \tag{12}$$

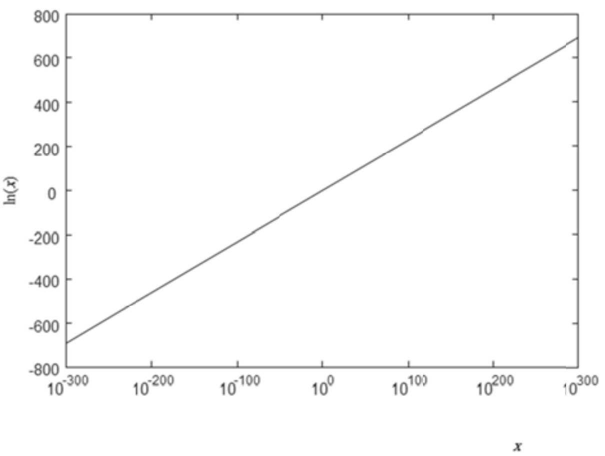

Fig. 2: Variation of x with $\ln(x)$ for very small and very large positive values

Accordingly, by taking the natural logarithm of both sides of Equation 4, the following equation can be obtained:

$$\ln\left(W(i)\right) = \ln\left(\prod_{j=1}^{n} f_j(i)\right) = \sum_{j=1}^{n} \ln\left(f_j(i)\right) \tag{13}$$

Using Equation 13, the multiplication of small numbers is avoided and the natural logarithm of the individual PDFs is summed instead; thus, numbers smaller than $d_{\min}$ will not be encountered by the software, except in the unlikely event that the value of the PDF for a given sample itself is less than $d_{\min}$.

It should be emphasized that Equation 7 will not be used to calculate the probability of failure in the proposed technique. Also, the result obtained by Equation 13 is not the weight index but rather the natural logarithm of the weight index. However, if W(i) > W(j), then by virtue of the strictly increasing property of the natural logarithm function illustrated in Fig. 2, ln(W(i)) > ln(W(j)). Accordingly, for the sake of sorting the samples according to the values of their weight indices, Equation 13 is appropriate. The samples are rather sorted according to the values of their natural logarithm of weight indices. In either case, the soring outcome is the same, but Equation 13 is numerically superior.

7. The proposed hybrid technique

The basic principle in the proposed hybrid technique lies in the fact that modified concepts from WASM are used to find the MPP. Then, the MPP is transformed to the standard normal space trough simple mathematical matrix operations as outlined earlier. Hence, the reliability index is calculated by Equation 11.

In WASM, the performance function is evaluated for all generated samples in order to calculate the probability of failure. As previously explained, the repetitive evaluation of performance functions that involves structural analysis of large structures imposes a significant computational burden over any simulation-based reliability analysis method. In order to remedy this problem, the approach for finding the MPP is modified herein such that the performance function is evaluated for a minimal number of samples.

The samples are arranged according to the value of ln(W(i)) in a descending order. Starting from the sample with highest ln(W(i)), and going through the samples one by one in the order of decreasing ln(W(i)), the index function, I(i), is calculated according to Equation 5. The process stops once the sample that provides a value of I(i) = 1.0 is reached. This sample is the MPP. It is the sample in the failure region having the highest weight index and thus the highest likelihood of failure. In this manner, the performance function is evaluated for only part of the samples. This surely saves the computational time that would have been required to evaluate I(i) for the remaining samples. This iterative process is illustrated in the flowchart in Fig. 3, which shows the algorithm of the proposed method. The steps of the algorithm can be summarized as follows:

1-The random variables and their properties are identified.
2-Proper intervals for each random variable in the problem are determined.
3-Samples for all random variables are generated in a random variable space. The uniform distribution can be used for generating these random samples.
4-The natural logarithm of the weight index is determined for each sample.
5-The samples are sorted in a descending order of ln (W(i)). Let q_1, q_2, ..., q_n be the indexes of the sorted samples in order.
6-Let t = 1.
7-The performance function is evaluated for the sample having the index q_t.
8-If $I(q_t) = 1.0$, the q_t^{th} sample is the MPP, i.e., $\mathbf{X}^* = S(q_t)$. Otherwise, t = t + 1 and repeat from Step 7.

9-Transform $\mathbf{X}^*$ into the standard normal space to obtain $\mathbf{U}^*$.
10-$\beta = ||\mathbf{U}^*||$

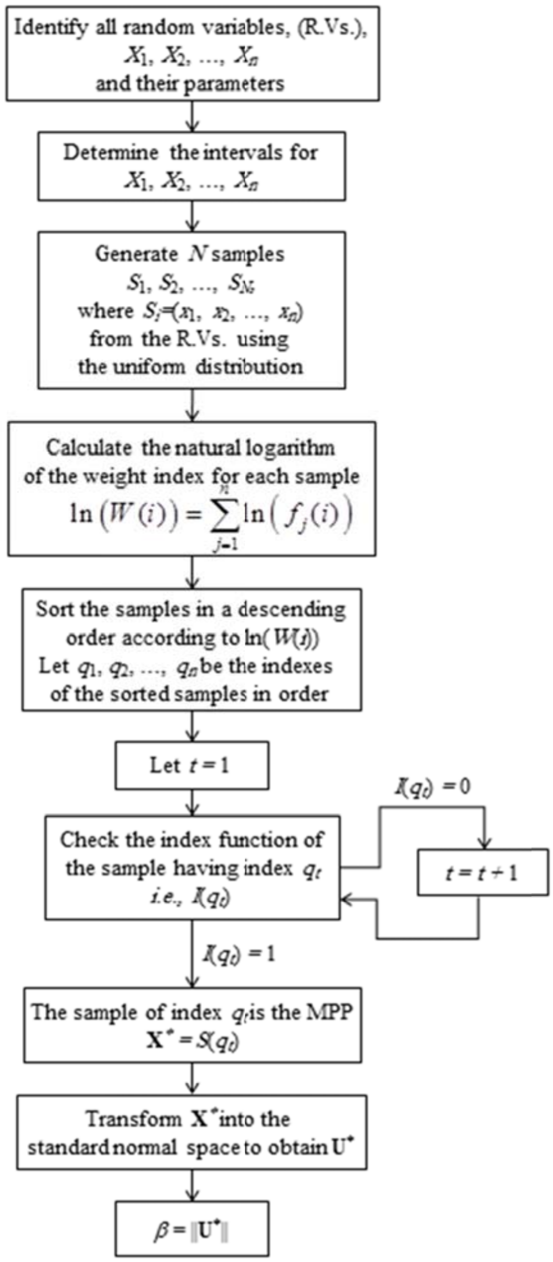

Fig. 3: Flowchart for the steps of the proposed hybrid technique

8. Examples

8.1. Parallel bar system

The parallel bar system shown in Fig. 4 is considered in this example in order to illustrate the efficiency of using Equation 13 for a problem with large random variables. In this investigation, the number of bars, n, is increased from 1 to 100. For each system created with a given number of bars, one random sample is generated from each of the random variables in the system using the uniform distribution. The random variables considered are the resistances of the bars. The mean of the bar resistances are taken as normally distributed random variables with means of 2.0×10^5 and standard deviations of 2.0×10^4. For the generated sample of each system, both the weight index and the natural logarithm of the weight index are calculated using Equations 4 and 13, respectively.

Fig. 5(a) shows the variation of the weight index as the number of bars increases, while Fig. 5(b) shows the variation of the natural logarithm of the weight index as the number of bars increases. As previously explained, the number of random variables in each created system is equal to the number of bars, n. It is clear from Fig. 5(a) that the weight index decreases rapidly as the number of bars increases. In fact, the weight index is less than 10^{-300} in a system containing 60 bars. As shown by the Figure, larger systems cannot be handled with the original weight index equation. Of course, the weight index can be different for different generated samples of the same system and also for systems with different properties. However, the conclusion drawn from this example is that the weight index equation can fail to provide results in large systems. The potential of numerical failure in Equation 4 grows in larger structures. This imposes a serious limitation on the original WASM regarding the size of the problems that can be handled by the WASM method.

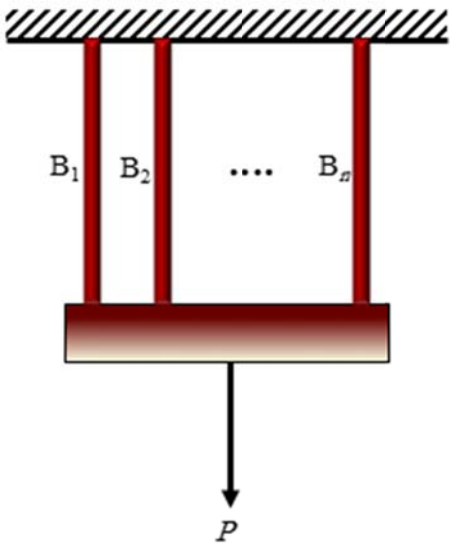

Fig. 4: Parallel bar system with *n* bars

It is evident in Fig. 5(b) that the logarithm of the weight index calculated by Equation 13 is a valid number for the systems with all the number of bars considered. The equation was capable of handling the size of the large systems that the original weight index equation couldn't.

8.2. Cantilever beam with two random variables

In this example, the proposed hybrid technique is illustrated on the cantilever beam example shown in Fig. 6. The details of this problem are adopted from Li et al. (2013) and Yang and Gu (2004). The performance function is established such that the tip displacement must not exceed the maximum allowed value, U_{max}. Therefore, the performance function is the difference between U_{max} and the tip displacement, and thus the function is given by:

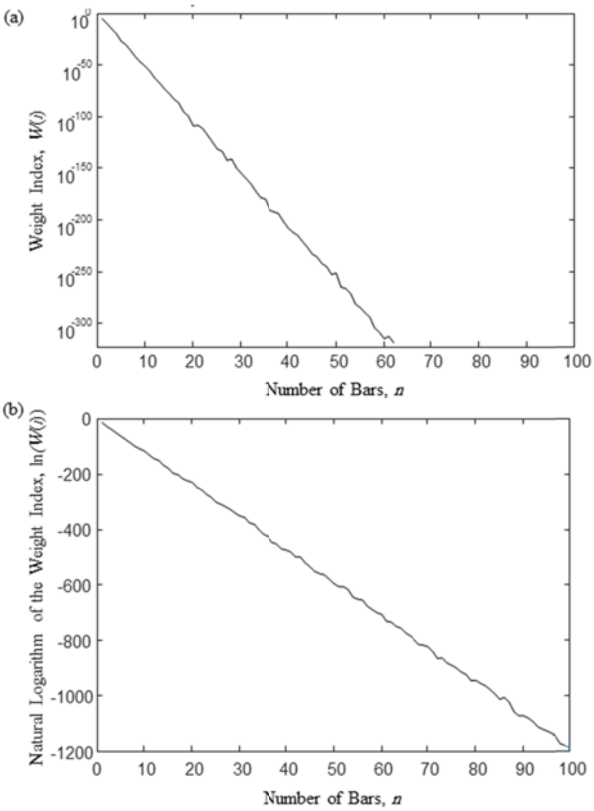

Fig. 5: Variation of (a) the weight index, $W(i)$, and (b) natural logarithm of the weight index, $\ln(W(i))$ with the number of bars, n

$$g(P_x, P_y) = U_{max} - \frac{4L^3}{Ewt}\sqrt{\left(\frac{P_x}{w^2}\right)^2 + \left(\frac{P_y}{t^2}\right)^2} \quad (14)$$

where U_{max} = 3 inch, E = 30×10^6 psi is the modulus of elasticity, L =100 inch is the length of the beam, and w = 2 inch and t = 4 inch are the width and height of its cross section, respectively. P_x and P_y are external forces treated as normally distributed random variables with means 500 lb and 1000 lb, respectively, and a standard deviation of 100 lb for both forces.

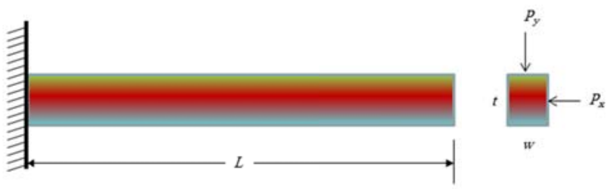

Fig. 6: Cantilever beam and its cross section

The problem is solved using the proposed technique with 1000 samples and the results are compared with those obtained from FORM. The details of the reliability analysis results for this cantilever beam example are shown in Table 1.

Fig. 7 shows the sample space for the random variables, the limit state function $g(P_x, P_y)=0$, and the MPP found using the proposed technique. Clearly, the proposed technique is capable of obtaining practically the same result that was obtained by FORM, in terms of reliability index and MPP. However, it is evident that FORM was able to find the solution with less number of performance function evaluations and less CPU time. It only took FORM 11 times to evaluate the performance function, whereas the proposed technique evaluated the performance function 95 times to find the first sorted sample that fails.

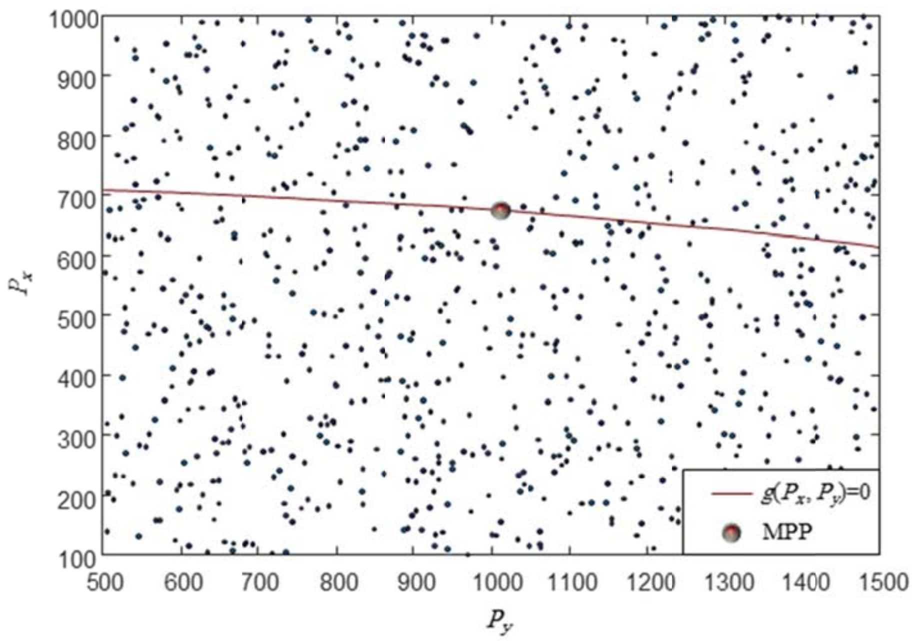

Fig. 7: Illustration of the MPP on the sample space for the cantilever beam example

If the WASM is used to solve this problem, a solution is reached only after evaluating the performance function for all 1000 samples. The proposed technique is clearly capable of solving this reliability analysis problem with a much less computational cost than required by the original

WASM. However, for problems with a small number of random variables, such as the present, FORM is expected to be computationally less time consuming. It should be noted, though, that Rashki et al. (2012) have shown that WASM is capable of solving problems that FORM and SORM couldn't handle. The proposed technique carries this same advantage.

Table 1: Details of the reliability analysis results for the cantilever beam example.

	Proposed Technique	FORM
Reliability Index, β	1.7448	1.7444
MPP, (P_x^*, P_y^*)	(673.9, 1013.6)	(673.7, 1016.4)
Number of Performance Function Evaluations	95	11
CPU Time (seconds)	1.9	0.2

8.3. An 82-bar truss bridge

The 82-bar truss bridge shown in Fig. 8 is adopted from Nakib (1997). The cross-sectional areas assigned to the bridge members are 3.0 in^2 for bottom and top chords and 1.5 in^2 for the vertical and diagonal elements. The modulus of elasticity of the truss members are treated as normally distributed random variables with coefficient of variation 0.1 and mean, 29000 ksi, respectively. Furthermore, the load P is treated as a normally distributed random variable with coefficient of variation 0.1 and mean of 30 kips. Accordingly, 83 random variables are treated in this problem.

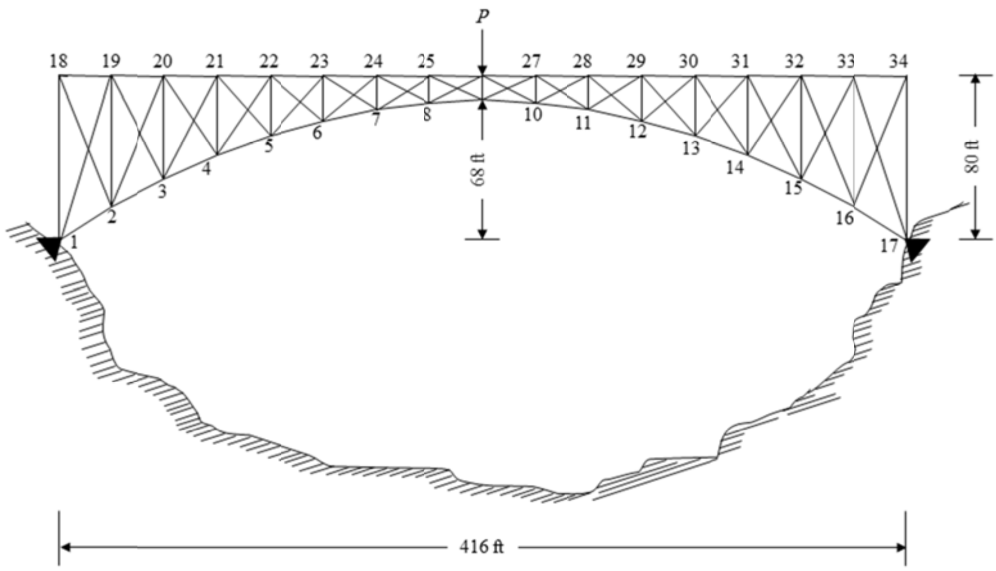

Fig. 8: 82-Bar truss bridge

The mode of failure considered is caused by excessive displacement of node 26. The performance functions associated with this mode of failure is written as follows:

$$g = U_{max} - U_{26} \tag{15}$$

where U_{max} = 0.5 ft is the maximum allowed displacement, and U_{26} is the absolute value of the displacement of node 26.

The problem was solved by the proposed technique with 1000 samples and the results are compared with those obtained from FORM. The forward finite difference is used in FORM for the gradient computations since U_{26} is determined by conducting a structural analysis of the truss bridge. The reliability index obtained is 4.8192 and 4.8178, from the proposed technique and FORM, respectively. Fig. 9 shows the number of structural analysis operations conducted in each method considered. As shown by the Figure, FORM required a significantly larger number of structural analyses than the proposed technique. This large number of operations is attributed in part to the numerical gradient computations with respect to the large number of random variables over several iterations of FORM. Furthermore, the original WASM requires conducting the structural analysis for all 1000 generated samples. It is clear from these results that the proposed technique harnessed the power of both dominant classes of reliability analysis methods and provided a platform for conducting structural reliability analysis of large structures in a significantly reduced computational cost.

9. Conclusions

In this paper, a hybrid technique for computing the reliability of large structures was presented. In this technique, modified concepts of the WASM are used to determine the MPP. The WASM concepts are modified in order to handle the problem of large random variables present in large structures and also in order to find the MPP in a computationally more efficient manner. The samples generated by the WASM are arranged according to the natural logarithm of the weight index in a descending order. The natural logarithm function is used to avoid the encounter of numbers smaller than those that computer software can handle due to limits in floating-point representations. Once the MPP is determined, it is transformed into the standard normal space. Hence, the reliability index is calculated in closed-form in the standard normal space.

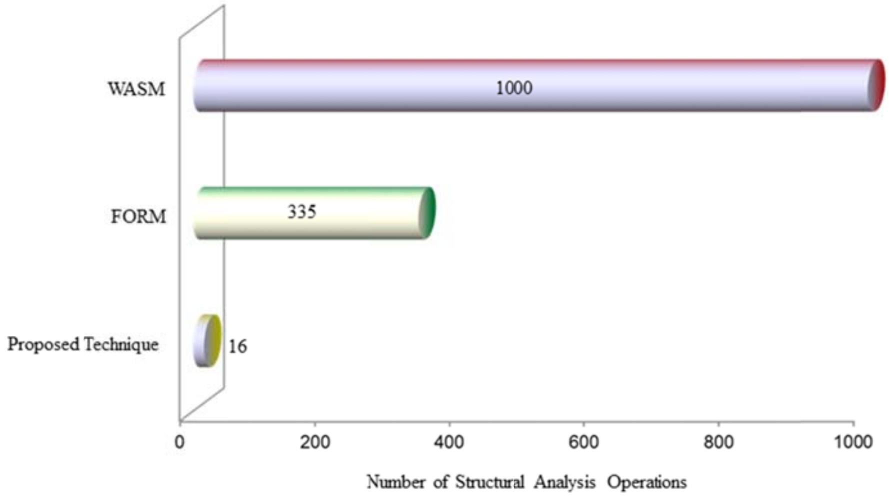

Fig. 9: Comparison of the number of structural analysis operations for the different methods used to solve the 82-truss bridge example

The proposed technique was shown to be capable of solving a reliability analysis problem with a small number of random variables. However, FORM was found computationally less time consuming in solving that example. However, the proposed technique exhibits the same advantage of WASM in that it is capable of solving problems that FORM and SORM can't handle. The approach was also tested on a truss bridge example. It was observed that the proposed technique required a significantly smaller number of structural analyses than did FORM. The computational efficiency of the proposed technique was evident.

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